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1- "A Design Study of a Numerically Controlled Frame Bending Machine", RINA. (UK-1965), Shama, M. A. and Miller, N. S., (90%)
2- "Plastic Bending of Short Mild Steel Beams", Bull. of the Faculty of Engineering, Alexandria University, (Egypt-1965), Shama, M. A. (100%)
3- "Behavior of Short Mild Steel Beams Bent into the Plastic Range of the Material" Bull. of the Faculty of Eng. Alexandria University, (Egypt-1967), Shama, M. A., (100%)
4- "The Impact of Application of a Numerically Controlled Plate Forming Machine on Shipyard Production" Bull of the Faculty of Engineering, Alexandria University, (Egypt-1968), Shama, M. A., 100%)
5- "Numerical Control of Plate Forming and Associated Problems", Shipbuilding and Shipping Record, Jan. (UK-1970), Shama, M. A., (100%)
6- " On the Calculation of Cold Forming Residual Stresses", Bull., Of the Faculty of Engineering, Alexandria University, Vol. XI. (Egypt-1970), Shama, M. A., (100%)
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FACULTY OF ENGINEERING

BEHAVIOUR OF SHORT MILD STEEL BEAMS BENT INTO THE PLASTIC RANGE OF THE MATERIAL

By

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Lecturer, Marine Engineering Department

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BEHAVIOUR OF SHORT MILD STEEL BEAMS BENT INTO THE PLASTIC RANGE OF THE MATERIAL

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INTRODUCTION

The paper describes some of the tests conducted to study the behaviour of rectangular section mild steel beams when they are bent nto the plastic range of the material. The main purpose was to investigate the 3-point bending method which is widely used in forming ship frames. As it was very expensive to carry out full scale tests on ship frames, it is believed that part of this investigation could be carried out on rectangular section mild steel beams using Universal Testing Machines.

The relationship between the applied load or bending moment and the different variables, in the elastic and plastic regions, were established experimentally. The variables considered are; the applied force or bending moment, the central deflection, the slopes at the supports, the curvature at midspan, the extreme fibre strain and the spring back in its different phases (in curvature, in central deflection and in slopes at the supports). The spring back is investigated in terms of the initial and final deformations as well as in relation to the applied load. In this paper a comparison is made between the experimental and theoretical results. The latter is based on the simple theory of elastic bending and is given in reference (1). It is shown in reference (1), that the relationship between the applied load or bending moment and the different variables depends entirely on the shape of the stress-strain diagram of the material used, namely yield stress, yield strain and rate of strain hardening. The true stress-strain diagram depends to a great deal on the stress state, residual stresses and strain aging.. Lloyds Register of shiping gives a wide scatter of the compositions of mild steel used in the shipbuilding industry. The combined effect of all these factors will cause a wide variation in the stress-strain diagram. Tests carried out shipbuilding steel can vary between 13.0 to 19.0 tons inch^2. This variation becomes very significant when other types of stresses are acting besides the normal stresses (i.e.) in the case of combined loading such as bending of deep sections.

Consequently, in order calculate the applied load or bending moment required to produce a certain amount of deformation (central deflection, curvature ... etc.), it is necessary to know the precise local stress-train diagram of the material at every section to be investigated. This is not practical, however, and the tests carried out were intended to assess, the degree of accuracy of the theoretical calculations which are based on an ideal stress-strain diagram.

It is to be noted that this experimental work has been carried out at Glasgow Universty, Naval Architecture Department.

The notations used are given at the end of the paper.

METHOD OF THREE-POINT BENDING

In practice, the method of the three-point bending is adopted for the shaping of ship frames in the cold state. There are two alternatives for load application :

a) A central moving ram and two fixed supports.

b) Two moving side arms and a central fixed support.

The three point bending produces axial thrust in addition to the transverse reaction at the supports. The magnitude of this axial thrust increases as the slope at the supports increases and it introduces additional normal and bending stresses, (see Fig. (1)). The effect of this side thrust should be taken into account when bending is in the plastic range since large deflections and slopes are obtained. However, it could be neglected when bending is in the elastic range, since the deflections and slopes are comparatively small.

It can be easily shown that, neglecting the moment of the friction forces at the supports, the bending moment may be calculated as follows

$$M = M_c - \left(\frac{W \tan \theta}{2}\right) \cdot \left(\frac{d}{2} - \Delta\right) \quad \ldots \quad \ldots \quad (1)$$

Where :

M = resultant bending moment at midspan.

$M_c = \frac{WL}{4}$ = maximum bending moment for a beam simply supported and centrally loaded.

θ = Slope at the suports.

W = Applied load at midspan.

d = Beam depth.

Δ = Central deflection with respect to the supports.

The values of the bending moments obtained from the experimental work are calculated from equation (1) so as to take into account the effect of side thrust.

EXPERIMENTAL WORK

a) *Test specimens* :

These were all mild steel beams of rectangular section having different $\frac{\text{depth}}{\text{span}}$ ratios, as given in table (1). The dimensions of the specimens were limited by the capacity of the available testing machines.

b) *Testing Machines* :

Two universal testing machines, were used for the laod application. The first has a total capacity of 50 tons and the other has a total capacity of 200,000 lbs (about 89.5 tons). The accuracy obtained was better than $\pm$ 1% for both machines.

c) *Measurements* :

1. *Strains* : The strains were measured by electric resistance strain gauges which are fixed on both sides of each beam. Tinsley equipments were used for strain measurements up to $\pm$ 0.5% strain. In addition, Phillips bridge G. M. 4571 was used for strain measurements up to $\pm$ 1.0% change of strain.

Table 1. Laboratory Test Beams.

Beam No.	Depth d inches	Thickness t inches	Span L inches	$\frac{L}{d}$	Momt. of Inertia I inch ^{4}
1	1.851	0.3180	12.0	6.483	0.1682
2	1.800	0.3455	12.0	6.666	0.1675
3	2.250	0.3500	9.0	4.000	0.3320
4	2.000	0.2515	9.0	4.500	0.1675
5	2.000	0.3000	9.0	4.500	0.2000
6	2.000	0.3000	9.0	4.500	0.2000
7	3.000	0.5000	12.0	4.000	1.1250
8	3.500	0.3750	16.0	4.571	1.3389
9	2.500	0.2500	16.25	6.500	0.3255
10	3.000	0.3770	13.25	4.416	0.8483

2. *Curvature* : The curvature at midspan was determined by using two strain gauges,one on each side of the longitudinal centre-line of the beam, allowance being made for the shift of the neutral axis. The following equation is used for curvature calculation.

$$\frac{1}{\rho} = \frac{\epsilon_c + \epsilon_t}{Z_c + Z_t}$$

Where :

ϵ_c = compressive strain at a distance Z_c from the longitudinal centre line of the beam.

Z_c = distance of the compressive strain ϵ_c from the longitudinal centreline of the beam;

ϵ_t = tensile strain at a distance Z_t from the centre line of the beam;

Z_t = distance of the tensile strain ϵ_t from the longitudinal centre-line see fig. (2).

$$\frac{1}{\rho} = \text{curvature at midspan attained by the neutral axis.}$$

Note :

If the curvature is cicular or parabolic, it can be shown from elementary geometry, that :

$$\frac{1}{R} \simeq \frac{8\Delta}{b^2}$$

Where :

$$\frac{1}{R} = \text{Curvature at the outer edge of the beam.}$$

$$\Delta = \text{Central deflection through a small distance } b.$$

Hence, using two dial gauges at a distance $\frac{b}{2}$ apart, such that one gauge is at midspan, the central deflection before and after spring back can be readily obtained, and the corresponding curvatures can be directly calculated.

3. *Slopes at the supports* : The slpes were deduced from the deflected profile using a dial gauge, which measured the rotation of the unbent part of the beam.

4. *Deflections* : These were measured relative to the machine bed at midspan and at other sections of the beam, by means of Mercer dial gauges which can read to + 0.0001 inch, see Fig. (3).

REST RESULTS

In the following analysis, a comparison is made between the experimental and the theoretical results. The latter are obtained from the simple theory of elastic and plastic bending.

The error percent is calculated as follows :

$$\textit{Error } \% = \frac{\text{Theoretical value-Experimental value}}{\text{Theoretical value}} \times 100$$

... Load-Deformation Results :

From these tests, it was found that the relationship between :

1. Bending moment & extreme fibre strain is as shown in Fig.(4).
2. Bending moment & slope at the supports is as shown in Fig. (5).
3. Load & central deflection is as shown in Fig. (6).

It is obvious that each curve could be divided into two regions, namely :

a) *Elastic Region* :

i. It was found that a certain amount of permanent set is inevitable after releasing the applied load. This is believed to be chiefly due to the residual stresses remaining after unloading.

ii. From table II, it is clear that the deviation of the experimental values from the corresponding theoretical values of EI could reach 36% and for EZ_e could reach 25%. This rules out the possibility of approximating 3 point bending by a beam simply supported and centrally loaded.

iii. In the case of M & ϵ relationship, the vales of M_{A1} are always less than the yield moment M_y, as given in Table III. These low values of M_{A1} are believed to be due to the effect of shear stresses, concentrated loading and possibly due to residual stresses.

b) *Strain Hardening Region* :

i. This region is represented by the curve BD in Fig. (4), which could be approximated by a straight line as shown in Fig. (13). See Apprndix (I).

Values of M_A, M_B and M_C as obtained from M & ϵ and M & Δ curves are given in Table (III) and are compared with M_y and M_p as calculated according to the simple theory of plastic bending. It is found that :

$$M_{A1} < M_y < M_{A2}$$

and $$M_{B1} < M_p < M_{B2}$$

These results indicate that M& ϵ relationship gives lower values for the forces and bending moments whereas M & Δ relationship gives higher values. The results obtained from M & Δ curves viz : M_{A2} & M_{B2} indicate clearly the existence of the strain hardening. The higher values of M_{A2} in comparison with M_{A1} implies that in spite of yielding at the extreme fibres, the central deflection still behave in an elastic fashion. This, in effect, justifies the statement given by Prager :

> "The elements of the greatest distance from the neutral axis are the first to reach the stress which brings them into the plastic region. But since this entry is accompanied by discontinuous increase in the extension (or compression) to which the nearer elements being still in the elastic state, cannot adapt themselves, the external elements do not as yet enter the plastic region, but pass the yield point still remaining elastic It is only after the bending moment has increased sufficiently for the whole transverse section to enter the plastic state that the transition to this state occurs instantaneously for all the elements of that section. The first plastic deformations shown by the so-called Lüders or Hartmans lines can not then be produced until the bending moment has reached the value that can be calculated by attributing to all the elements of the normal section the stress whose absolute values are equal to the yield point".

This statement was quoted by Volterra in reference (3).

ii. It was found experimentally that the ratio of $\frac{M_{B1}}{M_{A1}}$ varies between 1.4 to 1.6 with a mean value of 1.5 which is very close to the correponding theoretical value (i.e.) the shape factor as given by the simple theory of plastic bending.

$$\text{The shape factor} = \frac{\textit{Plastic modulus of the section}}{\textit{elastic modulus of the section}}$$

$$i.e. \quad \frac{M_p}{M_y} = \frac{Z_p}{Z_e} = 1.5 \text{ for a rectangular section.}$$

iii. The values of the bending moment M_{C2} which are obtained after neglecting the transition ragion were found to be very close to the values of the fully plastic moment Mp. From Table (III), it is clear that the ratio M_p/M_{C2} varies between 0.87 to 1.02 with a mean value very close to unity. In other words, the bending moment at the end of the elastic region in the modified $M \& \Delta$ cuve could be approximately calculated using the simple theory of plastic bending.

2. *Spring Back Results* :

a) The experimental results for the relationship between the applied load (or bending moment) and the spring back (in its different phases) are shown in Figs. (7) and (8). Each curve consists of two straight lines representing the elastic region *OA* and the strain hardening region *BD*.

b) In the elastic range it is shown that the theoretical slope of the "Load-recovered deformation" relationship, is always less than the experimental value, as given in Table (IV). This indicates the presence of a certain amount of permanent set which is rather difficult to compute since it dpends on the magnitude and distribution of the residual stresses existing before bending.

c) When bending is carried out in the strain hardening range the experimental values for the spring back are found th be larger than the corresponding theoretical values. This could be explained as follows :

When a beam is bent into the strain hardening region and then unloaded, the final stress and strain must comply with three conditions (4)

i. Strain distribution must remain linear before and after spring back to satisfy compatibility conditions. (Neglecting the effect of section distortion).

ii. The resultant normal force must be zero.

iii. The resultant bending moment of the residual stresses must also be zero.

In order to satisfy these conditions, the unloading line, in the strain hardening range, must be linear, as shown in Fig. (9). This indicates that the outer fibres. which were under tension, will be subjected to compressive residual stresses and vice versa. Consequently, the spring back of the outer fibres will be larger than the corrrespnding values obtained from a tension test, as shown by point "P" in Fig. (9). Hence, the spring back value depends on the magnitude of the applied bending moment, the geometry of the section, and shape of stress strain diagram. The latter becomes more difficult to determine precisely, when other types of stresses (radial stresses, stresses due to axial thrust, etc). are acting in addition to the normal stresses, as in the case of 3-point bending.

d) The exact calculation of any phase of the spring back (spring back in curvature, in slope, etc). is so far not possible due to the effects of the above mentioned factors. However, in the special case of W & δs relationship, the experimental results are reasonably close to the theoretical values when shear effect is taken into account. The maximum deviation was found to occur when the span/depth ratio is 4.0 as shown in Table (IV) for beams (3) and (7)

e) The effect of varying $\frac{span}{depth}$ ratio (i.e. $\frac{L}{d}$ ratio) from 4 to 7 on the central deflection is shown in Fig. (10). It is clear that there is good agreement between theoretical and experimental reults at some points. It is believed that the scatter of some of the points is mainly due to measuring errors and possibly due to using short beams having $\frac{L}{d}$ = 4.0 which may be subjected to a very complex stress system.

3. *Results Obtained From Deformations Before and After Spring Back.*

It was found that the relationship between the central deflection, slope ... etc., before and after spring back is as shown in Figs. (11) and 12. Each curve could be divided into two regions, namely :

i. *Elastic and Elasto-Plastic Region* :

This region is represented by the transition curve OA. As deformations taking place in this region are relatively small, we shall only consider the strain hardening ragion, Figs. (11), (12).

ii. *Strain Hardening-Region* :

This ragion is represented by the straight line AB. The results are shown in Table *(V)* for the Δ&δ curve. It is shown that there is good agreement between the theoretical and experimental results for the slope of the line AB whereas there is less agreement btween the experimental and calculated central deflections (i. e.) Δ_C and Δ_{CC}. (See Appendix II).

As for the $\frac{1}{\rho_1}$ & $\frac{1}{\rho_2}$ relationship the slope of the straight line AB, representing the strain hardening region is given by $\frac{E}{E-\tan\alpha} \cong 1.01$ which is very close to unity. The exprimental slope was found to vary from 1.0—1.03 which indicates good agreement, see appendix III. It is concluded therefore that assuming linear strain hardening gives reasonable correlation between theoretical and experimental results for the deformations before and after spring back.

Conclusions :

From the previous test analysis and results, it is concluded that:

1. Due to the variable nature of the steel used in the shipbuilding industry, it is impossible to calculate the forces required to form plates or sections to any specific shape.

2. Because of the inability to predict spring back precisely, trial and error is the only solution to from a plate or section into a specific shape.

3. If the bending machine used for forming plates or sections is to be automated, the control system must involve a servo-mechanism to overcome the variation in spring back.

4. The simple theory of elastic and plastic bending cannot be applied for the cases of relatively short beams having span depth ratios of from 4 to 7.0.

5. The 3-point bending method cannot be approximated by a beam simply supported and centrally loaded.

6. The effect of strain hardening on the different calculations was considerable. The simple theory of plastic bending cannot be used to explain the behaviour of relatively short beams bent well into the strain hardening range of the material. The deviation of the experimental results from the calculated values for the applied force (or moment) was found to be in the range of 15%—35%.

7. It was rather difficult to meassure curvatures whether indirectly by means of strain gauges (due to the high strain values which were out of the range of the available strain gauges) or directly by dial gauges

(due to the relatively large errors resulting from measuring very small deflections). Had these beams been of larger spans it would have been more accurate to measure the curvatures by means of dial gauges.

8. It is belived that in order to have better correlation between theoretical and experimental results, it is necessary to investigate the mechanics of plastic bending with particular reference to starin and stress distribution in the plastic zone.

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$\tan \gamma_1$ = slope of the elastic line in the $M \& \epsilon$ curve for the outermost fibres.

$\tan \gamma_2$ = slope of the elastic line in the $M \& \frac{1}{\rho}$ curve when calculated from :

$$\frac{1}{\rho} = \frac{\epsilon_t + \epsilon_c}{Z_t + Z_c}$$

$\tan \gamma_3$ = Theoretical slope of the elastic line in the $M \& \theta$ relationship (1)

$$(i.e.) \tan \gamma_3 = \frac{EI}{\frac{L}{4} + \frac{6.24\, I}{AL}}$$

$\tan \gamma_4$ = Experimental slope of the elastic line in the $M \& \theta$ curve.

$\tan \gamma_5$ = Theoretical slope of the elastic line of the $W \& \Delta$ relationship (1)

$$(i.e.) \tan \gamma_5 = \frac{4Eb}{\left(\frac{L}{d}\right)^3 + 3.12\left(\frac{L}{d}\right)}$$

$\tan \gamma_6$ = Experimental slope of the elastic line of $W \& \Delta$ curve.

M_{A1} = Bending moment at the end of the elastic line in the $M \& \epsilon$ curve.

M_{A2} = Bending moment at the end of the elastic line in the $M \& \Delta$ curve.

M_y = Yield moment.

M_p = Fully plastic moment.

Δ = Central deflection before spring back.

δ = Central deflection after spring back.

δ_s = Spring back in central deflection.

θ_s = Spring back in slope at the supports.

M_e = Bending moment attained by the alastic core.

M_o = Intercept on the M-axis in M & ϵ relationship.

M_{B1}, M_{B2} = Bending moment at the beginning of the strain hardening range in the M & ϵ and M & Δ relationships.

M_{C1} = Bending moment at the end of the elastic line in the modified M & ϵ relationship.

M_{C2} = Bênding moment at the end of the elastic line in the modified M & Δ relationship.

$\tan\ \alpha_1$ = Experimental slope of the M & $\frac{1}{\rho_s}$ curve.

$\tan\ \alpha_2$ = Theoretical slope of M & θ_s relationship (1)

$$\text{i.e. } \tan \alpha_2 = \frac{EI}{\frac{L}{4} + \frac{6.24I}{AL}}$$

$\tan\ \alpha_3$ = Experimental slope of the M & θs curve.

$\tan\ \alpha_4$ = Theoretical slope of W & δ_s relationship

$$i.e.\ \tan \alpha_4 = \frac{4Eb}{\left(\frac{L}{d}\right)^3 + 3.12\left(\frac{L}{d}\right)}$$

$\tan\ \alpha_5$ = Experimantal slope of W & δ_s curve.

$\tan\ \psi_1$ = Experimental slope of Δ & δ curve in the plastic range.

Δ_0 = Intrecept of Δ & δ curve with the Δ — axis.

Δ_c , Δ_{cc} = Experimental and calculated central deflections at the load W_C.

tan ψ_2 = Calculated slope of Δ & δ curve (see appendix (1).

tan β = Experimental slope of the strain hardening range in the W & Δ curve.

Note : All the above theoretical expressions are given in detail in reference (1).

TABLE (II) Elastic Region Results

Beam No.'	1	2	3	4	5	6
d	1.851	1.800	2.250	2.000	2.000	2.000
L/d	6.483	6.666	4.000	4.500	4.500	4.500
EZ_e	2380	2370	3865	2193	2620	2620
tan γ_1	2550	2650	3880	2745	—	2820
% error	—7.14	—11.8	—0.389	—25.2	—	—7.63
EI	2205	2195	4350	2193	2620	2620
tan γ_2	2500	2660	5000	3000	—	2950
% error	—13.4	—21.2	—14.95	—36.8	—	—12.6
tan γ_5	128.2	129.0	240.3	125.5	149.70	149.7
tan γ_6	97.8	111.2	160.0	93.30	106.80	135.0
% error	+23.7	+13.8	+33.33	+25.65	+28.70	+9.83

Beam No.	1	2	3	4	5	6	Notes
M_{A1}	2.390	2.375	4.290	1.950	2.80	3.150	From M.& ϵ
M_{B1}	3.750	3.900	6.100	3.150	4.20	4.500	Curves
M_{C1}	3.230	2.730	5.360	2.475	4.00	3.725	
M_{B1}/M_{A1}	1.571	1.642	1.422	1.617	1.56	1.428	
M_{A2}	4.00	4.500	5.800	3.100	3.80	4.050	From M. & Δ
M_{B2}	5.20	5.400	7.700	4.500	5.50	5.700	
M_{C2}	4.78	5.000	7.050	3.910	4.90	4.790	Curves
Z_e	0.1817	0.1811	0.2951	0.1675	0.200	0.200	
M_y	2.9100	2.8980	4.7200	2.680	3.200	3.200	
Z_p	0.2727	0.2713	0.4420	0.2512	0.300	0.300	
M_p	4.3650	4.3400	7.0700	4.0200	4.800	4.800	
M_p/M_{A2}	1.0910	0.9650	1.2200	1.2980	1.263	1.185	
$M_p/_{B2}$	0.8400	0.8030	0.9180	0.8950	0.873	0.842	
M_p/M_{C2}	0.9140	0.8680	1.0030	1.0280	0.980	1.002	
M_y/M_{A1}	1.2150	1.2150	1.1000	1.3730	1.142	1.015	

Note : The yield stress σ_y = 16.0 tons/inch^2.

TABLE (IV) SPRING BACK RESULTS

Deviation between theoretical and experimental results

Beam No.	EI	$\tan \alpha_1$	% error	$\tan \alpha_2$	$\tan \alpha_3$	% error	$\tan \alpha_4$	$\tan \alpha_5$	% error
1	2205	—	—	900	1167	—29.7	128.2	128.5	—0.234
2	2195	—	—	902	1190	—31.95	129.0	121.7	+6.75
3	4350	5110	—17.47	1710	2340	—36.80	240.3	202.0	+15.95
4	2193	2660	—21.30	885	1192	—34.70	125.50	116.6	+7.10
5	2620	2600	+0.763	1056	—	—	149.70	193.2	+7.02
6	2620	3050	—16.4	1056	1263	—19.60	149.70	148.0	+1.136
7	—	—	—	—	—	—	343.0	297.0	+13.40
8	—	—	—	—	—	—	179.5	182.8	—1.84
9	—	—	—	—	—	—	44.45	47.3	—6.41
10	—	—	—	—	—	—	198.30	213.5	—7.41

TABLE (V) PERMANENT DEFORMATION RESULTS

Beam No.	$\tan\psi_1$	$\tan\beta$	$\tan\psi_2$	Δo	Δc	Δcc	% error
1	1.04	4.445	1.036	0.016	0.0198	0.0168	—0.386
2	1.034	4.170	1.034	0.018	0.0200	0.0172	0.000
3	1.040	8.420	1.038	0.015	0.0190	0.0129	—0.192
4	1.042	3.530	1.030	0.016	0.0175	0.0138	—1.163
5	1.035	4.740	1.034	0.0148	0.0200	0.0145	—0.097
6	1.035	4.680	1.032	0.014	0.0155	0.0142	—0.291
7	1.230	—	—	0.0335	—	0.0364	—
8	1.045	8.70	1.051	0.028	0.032	0.0284	+0.571
9	1.042	1.820	1.042	0.0800	0.0860	0.0878	0.006
10	1.015	1.640	1.008	0.0560	0.0630	0.0610	—0.69

Bending Moment-Extreme Fibre Strain Relationship.

Tests were carried out on standard specimens to obtain a mean value for th yield stress σ_y, and rate of strain hardening tan α.

It was found that :

$$\sigma_y = 16.0 \text{ tons/inch}^2$$

$$\tan \alpha = 150\text{—}160 \text{ tons/inch}^2$$

$$E = 13400 \text{ tons/inch}^2$$

The bending moment M is calculated from :

$$M = M_e + (\sigma_y - \epsilon_y \tan \alpha)\, Z_p + \frac{I_p}{\rho} \tan \alpha$$

$$= M_e + M_p - \epsilon_y \,.\, Z_p \tan \alpha + \frac{I_p}{\rho} \tan \alpha$$

M_e could be neglected without an appreciable error.

$$M_p = Z_p \,.\, \sigma_y$$

$$Z_p = KZ_e = 1.5\, Z_e$$

K = Shape afactor = 1.5 for rectangular sections

$$M_p = 24\, Z_e$$

$$\epsilon_y = \frac{\sigma_y}{E}$$

$$\tan \alpha \cong 0.011\, E \text{—} 0.012\, E$$

I_p = moment of inertia of the plastic portions of the section.

$\therefore$ I_p = I = moment of inertia of the section (when neglecting elastic core).

$$\frac{1}{\rho} = \frac{\epsilon_1}{d/2}$$

where ϵ_1 = extreme fibre strain

d = depth of beam

$$\therefore \quad M = (23.7 + 160.8\, \epsilon_1)\, Z_e$$

which is a straight line having a slope of 160.8 Z_e and an intercept of 23.7 Z_e.

APPENDIX II

Slope of Δ - δ curve :

From Fig. (13), we have :

$$\Delta = \delta + \delta_s$$

$$= \delta + \frac{W}{\tan \gamma_5} \qquad \ldots \ldots (1)$$

But $W = W_c + (\Delta - \Delta_c) \tan \beta \qquad \ldots \ldots (2)$

From (1) and (2) we have :

$$\Delta = \delta \tan \psi_2 + \Delta_c \qquad \ldots \ldots (3)$$

where $\tan \psi_2 = \dfrac{\tan \gamma_5}{\tan \gamma_5 - \tan \beta}$

APPENDIX III

Relationship between the two curvatures before and after spring back.

The bending moment-curvature relationship is :

$$M = M_p + I \tan \alpha \left(\frac{1}{\rho_1} - \frac{1}{\rho_2} \right) \qquad \ldots \ldots (1)$$

The beding moment-spring back relationship is :

$$\frac{M}{EI} = \frac{1}{\rho_1} - \frac{1}{\rho_2} \qquad \dots \dots (2)$$

From (1) and (2), the relationship between $\frac{1}{\rho_1}$ and $\frac{1}{\rho_2}$ is given by :

$$\frac{1}{\rho_1} = \frac{E}{E - \tan\alpha} \frac{1}{\rho_2} + \frac{1}{\rho_p} \qquad \dots \dots (3)$$

This is *a* linear relationship having :

$$\text{Slope} = \frac{E}{E - \tan\alpha}$$

$$\simeq \frac{13400}{13400-150} \simeq 1.01$$

Equation (3) becomes

$$\frac{1}{\rho_1} \simeq 1.01 \frac{1}{\rho_2} + \frac{1}{\rho_p} \qquad \dots \dots (4)$$

where $\frac{1}{\rho_p}$ = curvature attained when the fully plastic moment is applied.

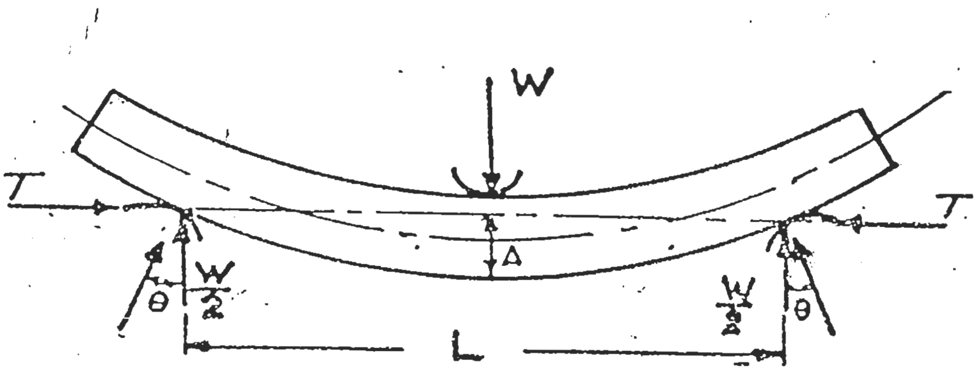

FIG. (1).

THREE - POINT BENDING

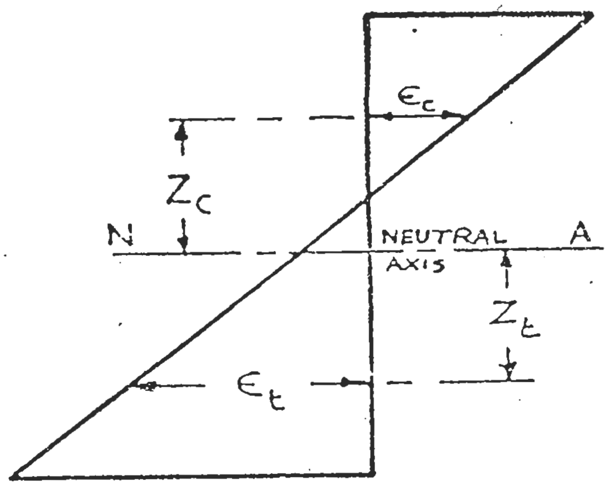

FIG. (2) STRAIN DISTRIBUTION

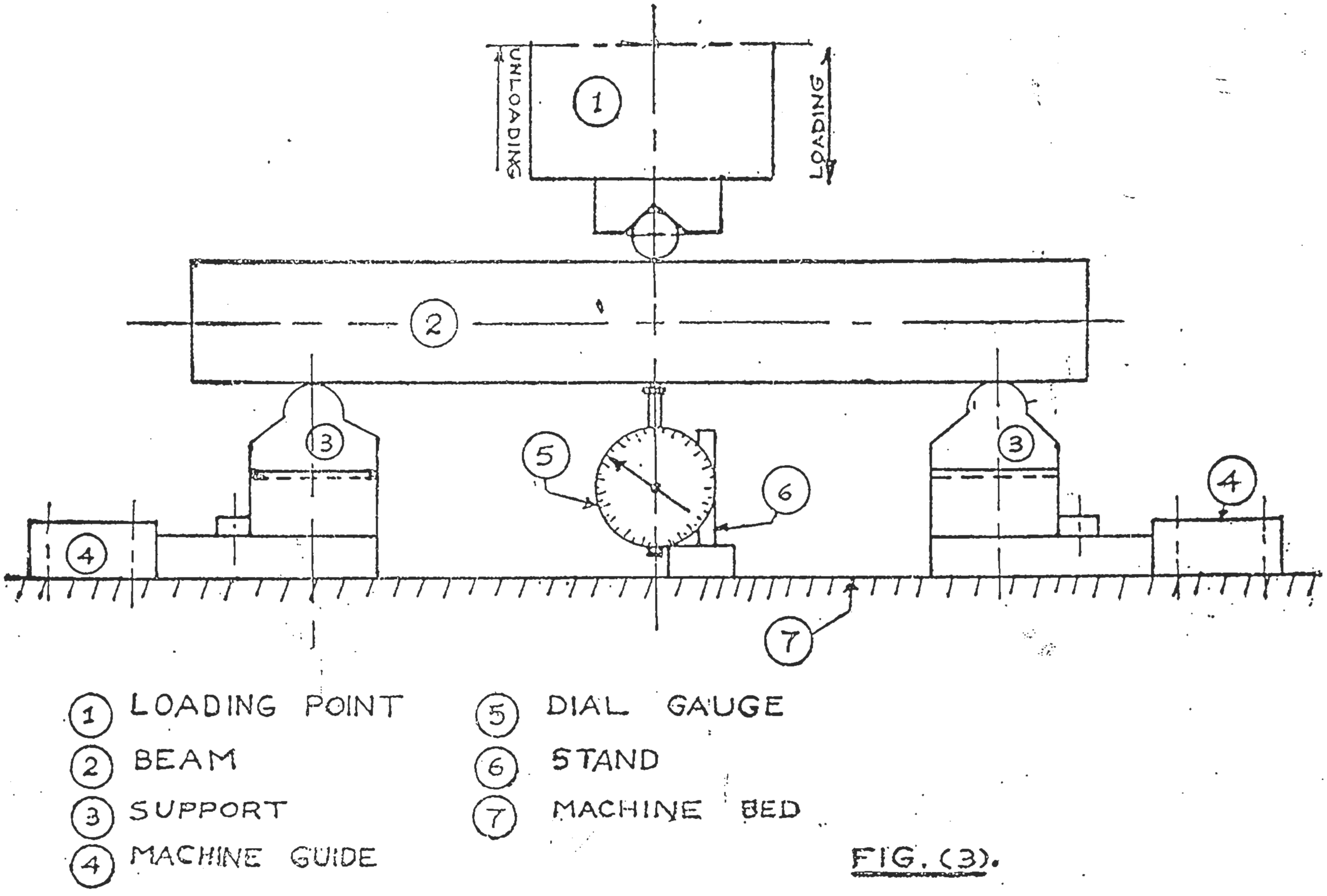

FIG. (3).

Bending Arrangement

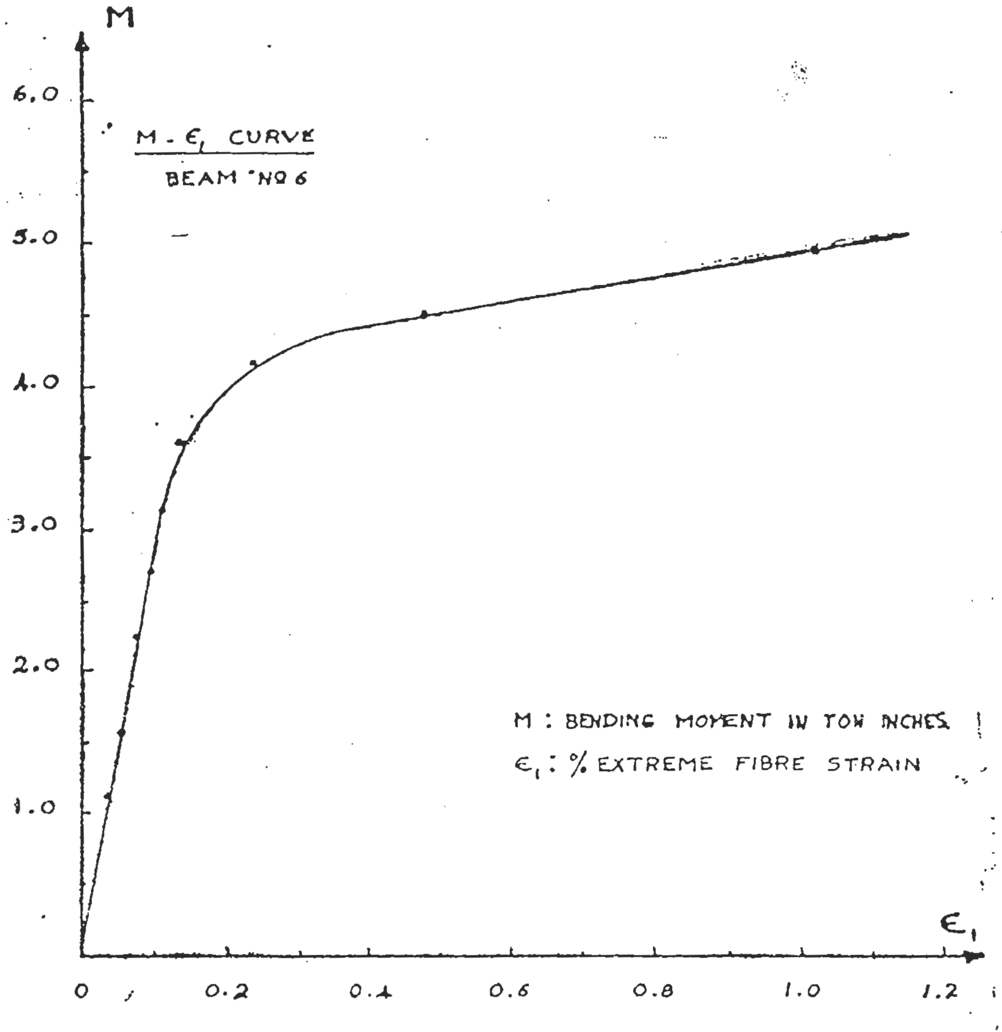

Fig. (4).

Bending Moment - Strain Curve

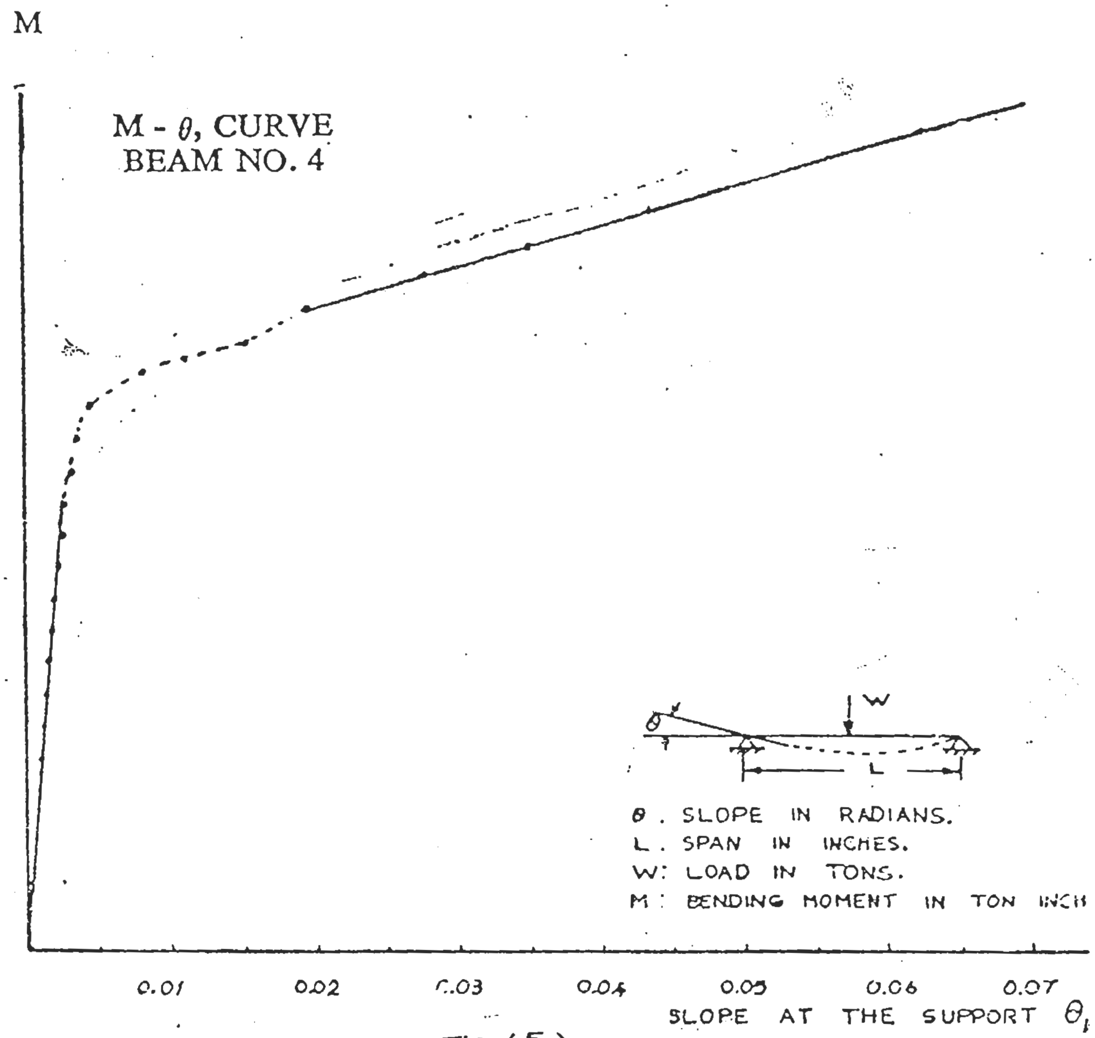

Fig. (5).

Bending Moment - Slope Curve

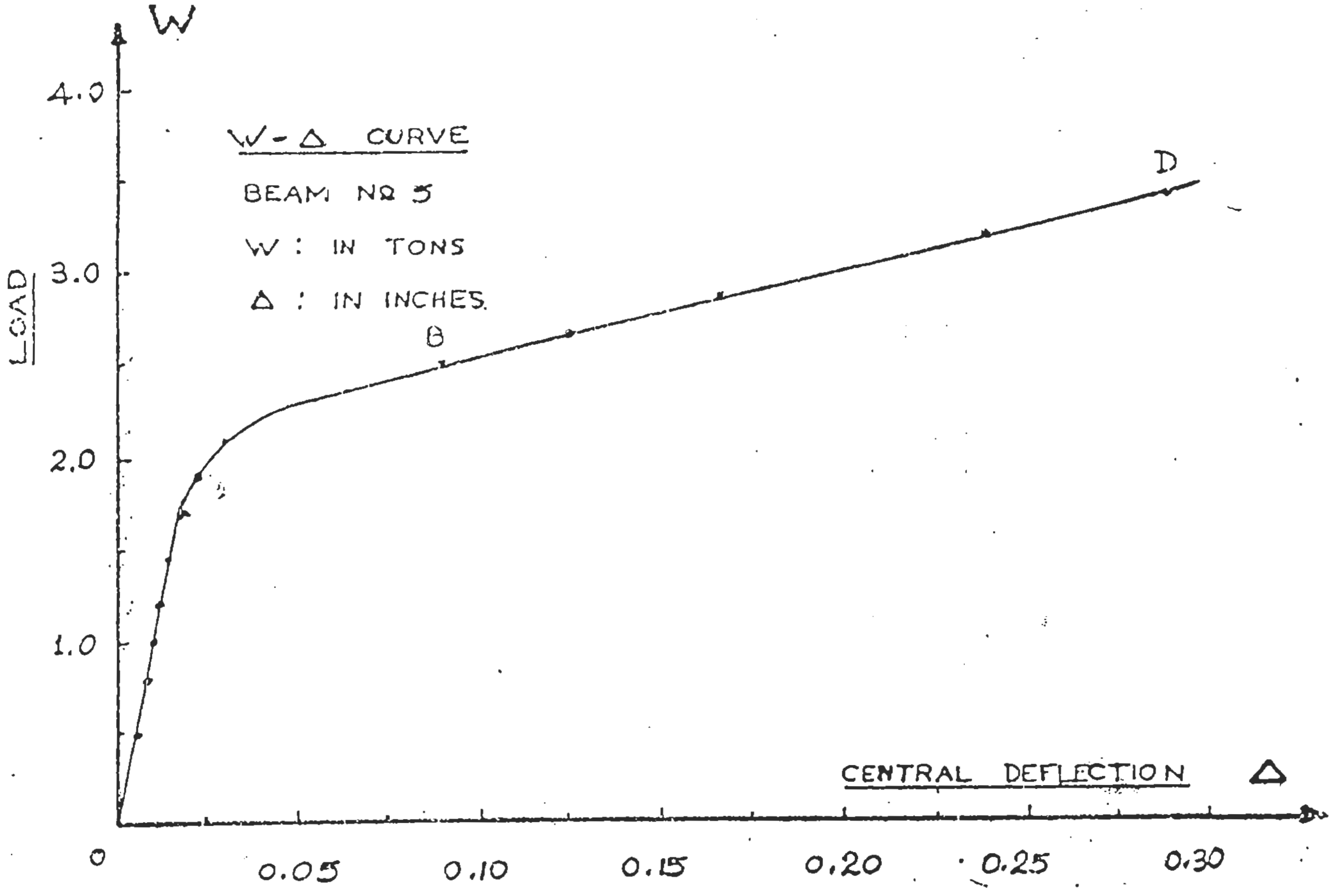

Fig. (6).

Load - Deflection Curve

Load - Deflection Curve

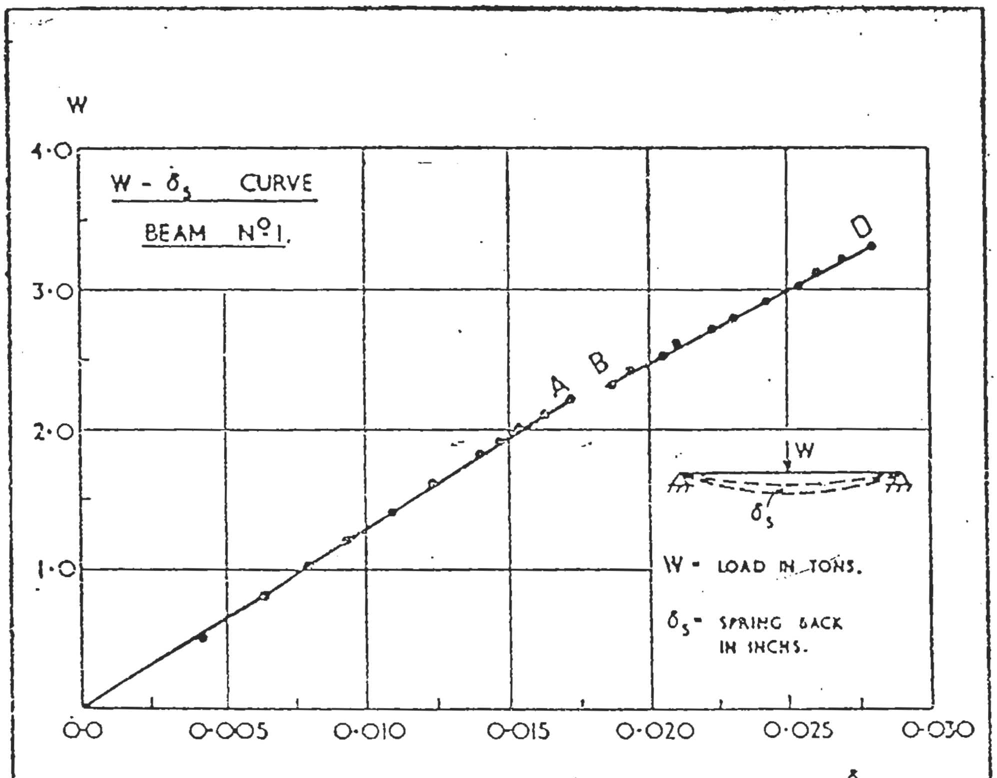

FIG. (7)

Load - Spring Back Curve

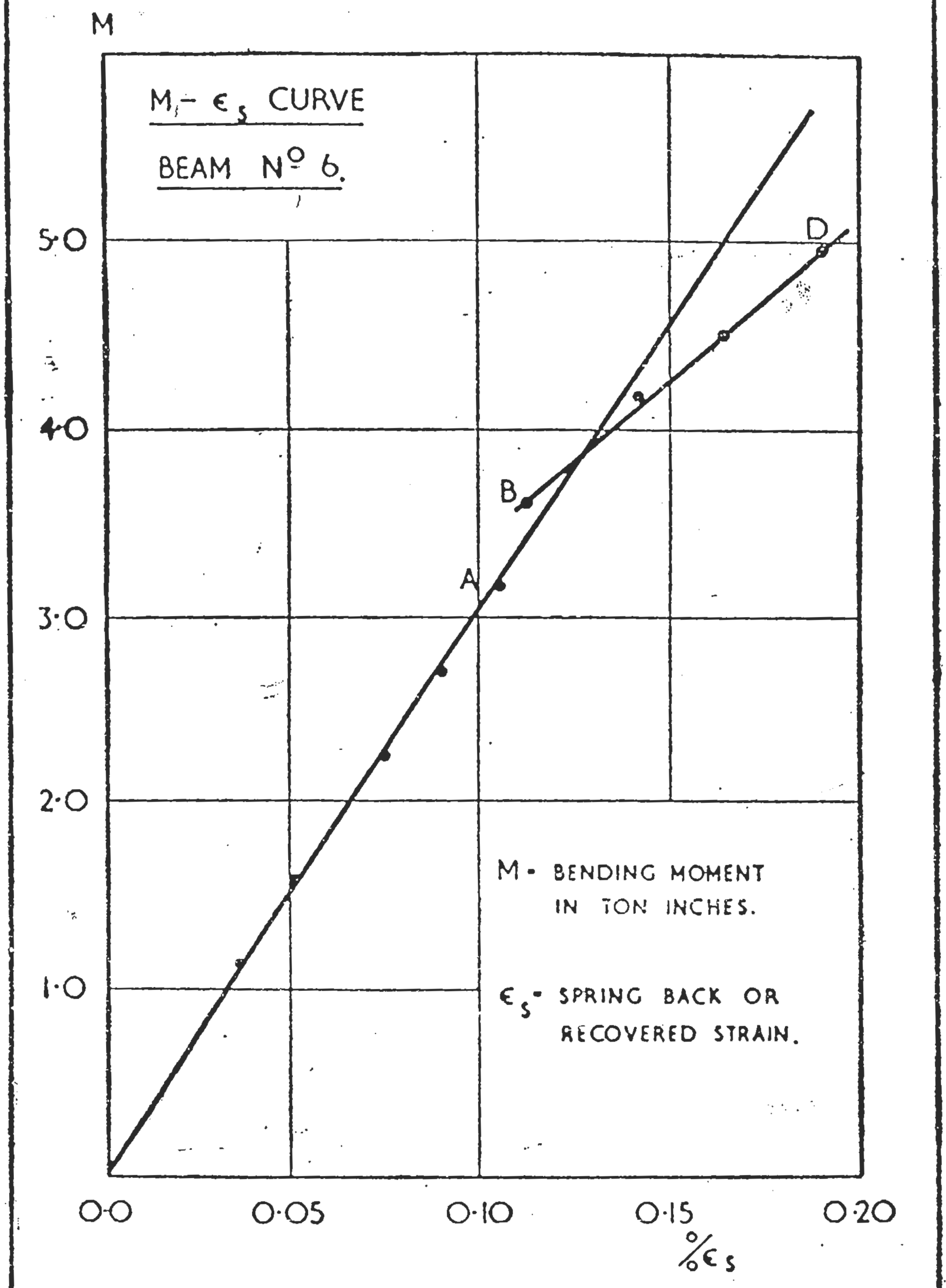

FIG.(8)

Bending Moment - Recovered Strain Curve

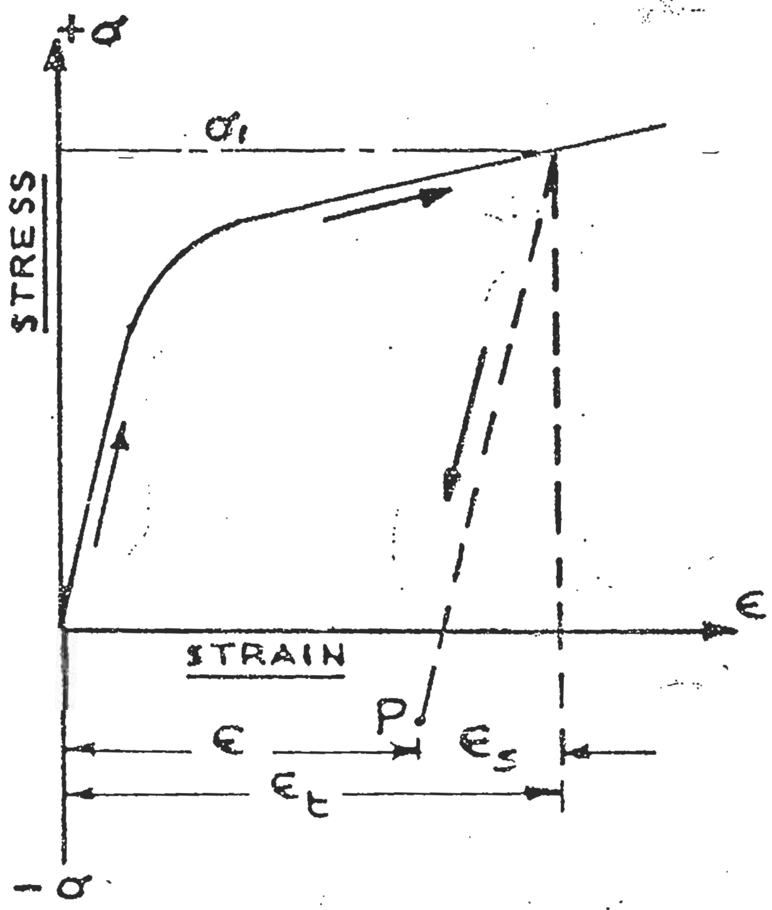

Fig.(9).

Recovered Strain in Bending

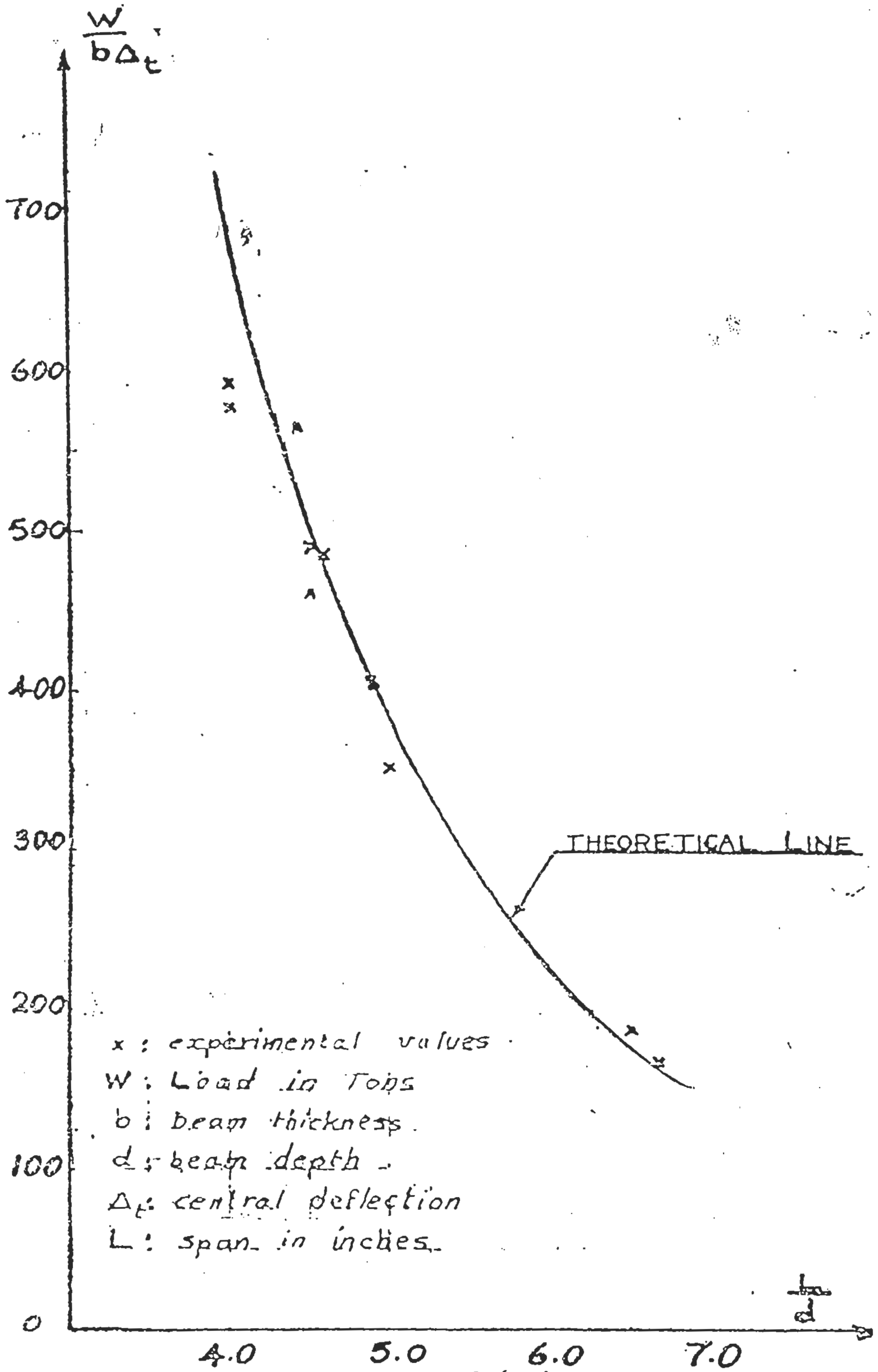

FIG. (10)

Effect of Increasing $\frac{\text{SPAN}}{\text{DEPTH}}$ Ratio

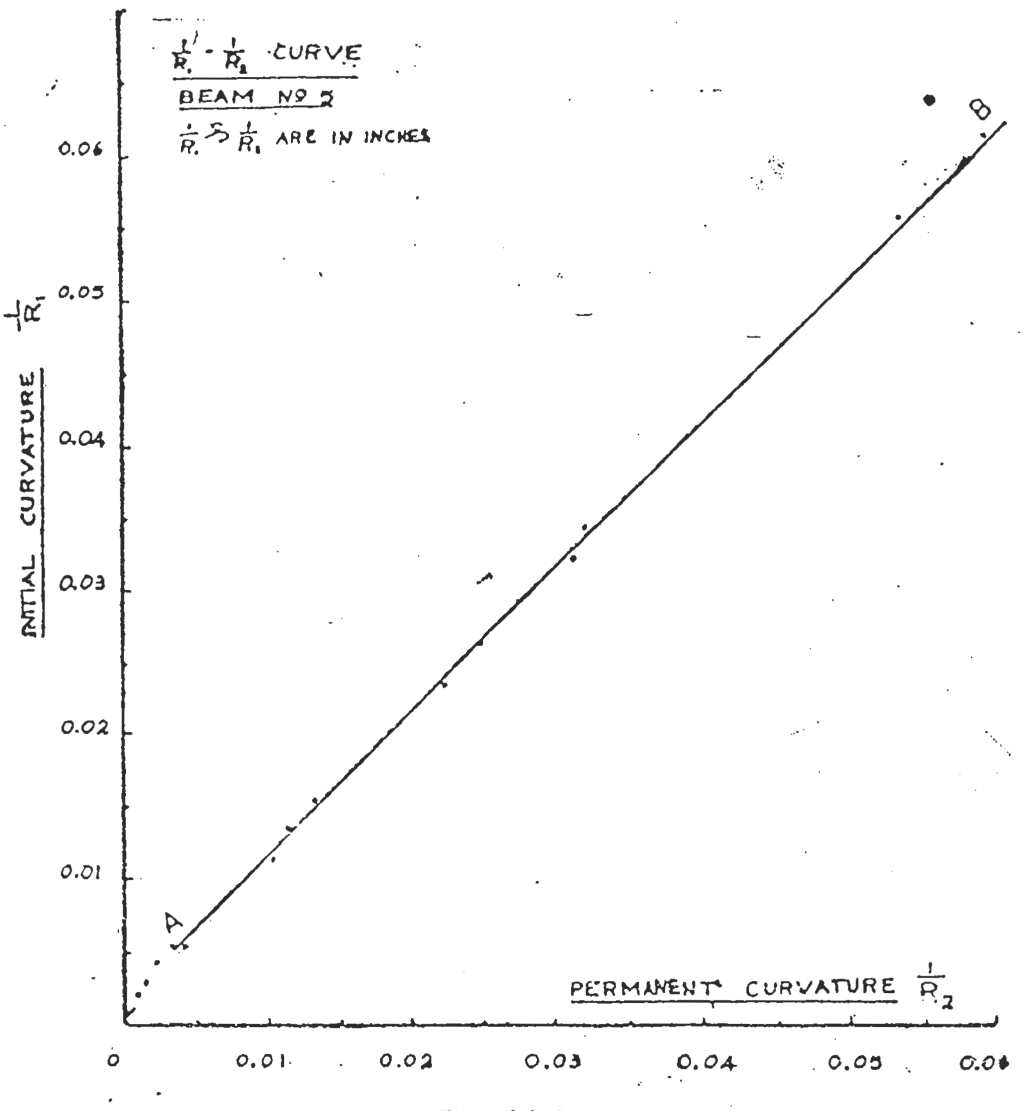

Fig. (11)

Relationship Between the Curvature Before and After Spring Back

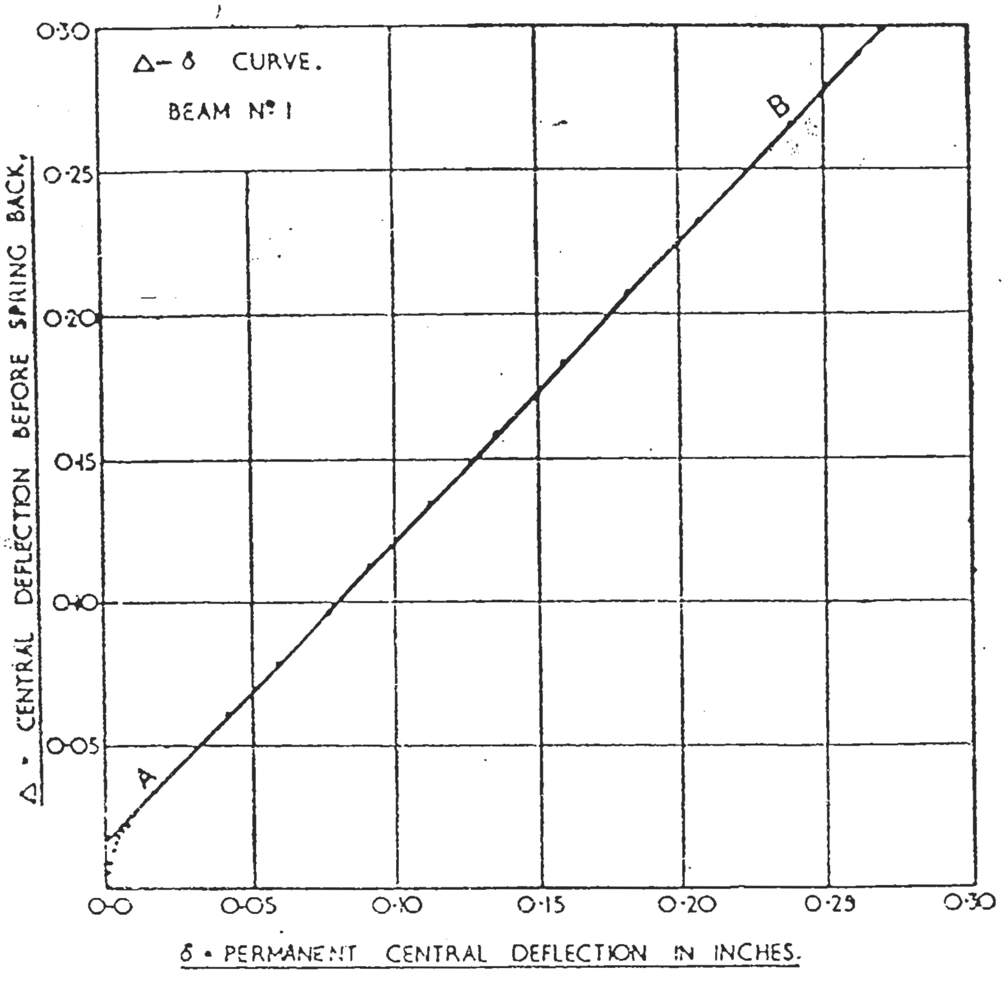

FIG. (12).

Relationship Between the Deflections Before aud After Spring Back.

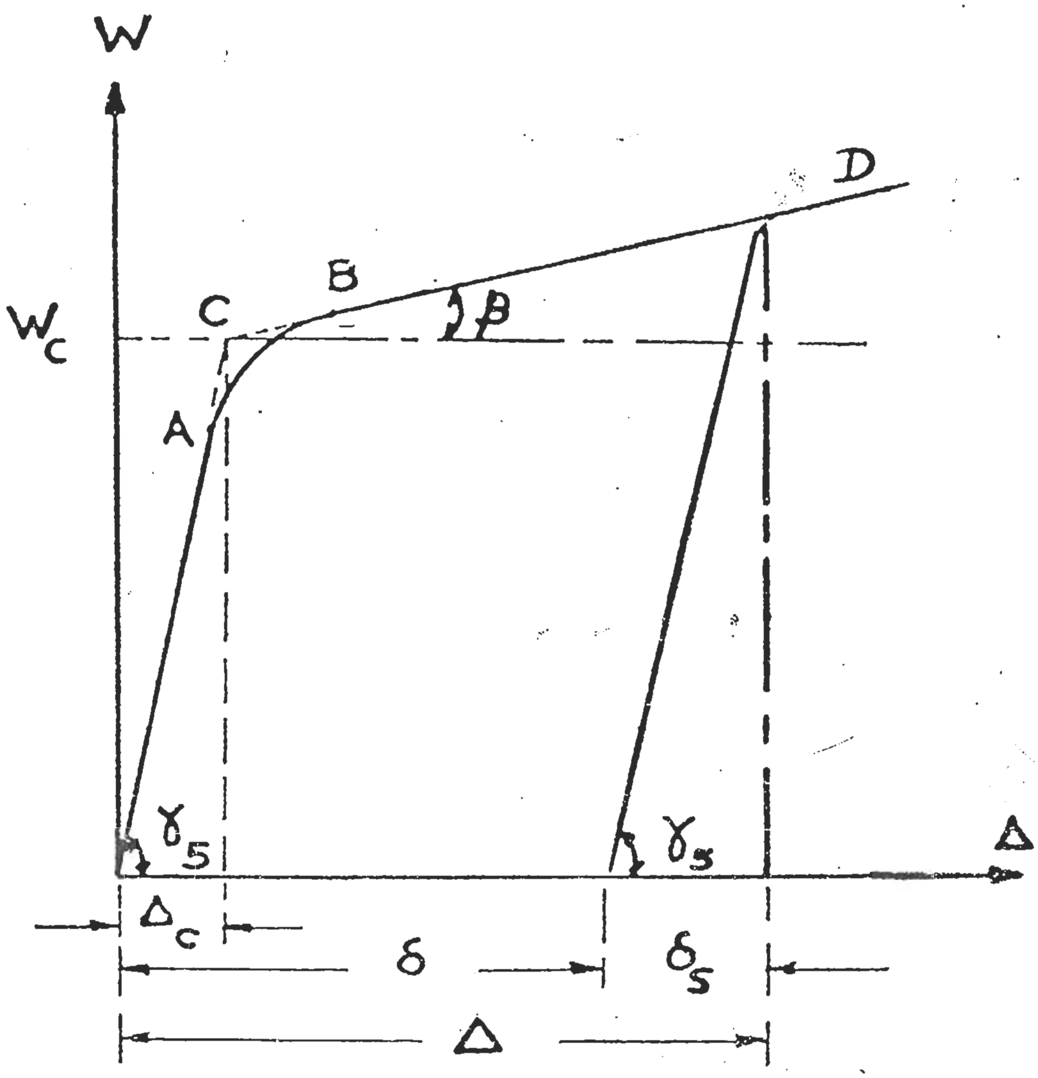

Fig. (13).

Load - Deflection Curve